The dynamic relationship between the federal funds rate and the Treasury bill rate: An empirical investigation

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Abstract

This article examines the dynamic relationship between two key US money market interest rates—the federal funds rate (FF) and the 3-month Treasury bill rate. Using daily data over the period from 1974 to 1999, we find a long-run relationship between these two rates that is remarkably stable across monetary policy regimes of interest rate and monetary aggregate targeting. Employing a nonlinear asymmetric vector equilibrium correction model, which is novel in this context, we find that most of the adjustment toward the long-run equilibrium occurs through the FF. In turn, there is strong evidence for the existence of significant asymmetries and nonlinearities in interest rate dynamics that have implications for the conventional view of interest rate behavior.
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1. Introduction

Interest rates tend to move together. The nature of the forces that bind interest rates together is not fully understood, however. This paper investigates the

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relationship between two important short-term interest rates in US financial markets—the overnight federal funds rate (FF) and the 3-month Treasury bill (or T-bill) rate (TB). The importance of these two interest rates is widely recognized. The Federal Reserve (Fed) implements monetary policy by targeting the effective FF; the 3-month TB is the preeminent default-risk-free rate in the US money market, and is often used by researchers to proxy the risk-free asset whose existence is assumed by much conventional finance theory. Given their importance and visibility, it is not surprising that these interest rates have been studied extensively in economics and finance (see, inter alia, Hall et al., 1992; Taylor, 1993; Rudebusch, 1995, 1998; Anderson, 1997; Enders and Granger, 1998; Stock and Watson, 1999).

A number of authors have argued that the FF and the TB move together because they are linked by the expectations hypothesis (EH) (e.g., inter alia, Cook and Hahn, 1989; Goodfriend, 1991; Poole, 1991; Rudebusch, 1995, 2001; Woodford, 1999). That is, it is assumed that the TB is equal to the market's expectation for the FF over the term of the TB plus a risk premium.

In the last 15 years or so, several influential empirical studies have focused on testing the EH of the term structure of interest rates using cointegration and equilibrium correction models (ECMs) (see Engle and Granger, 1987; Stock and Watson, 1988; Campbell and Shiller, 1991; Hall et al., 1992; Engsted and Tanggaard, 1994). Empirical support for the EH is generally weak (e.g., Campbell et al., 1997). This is particularly true when the short-term rate is the effective FF (see, inter alia, Simon, 1990, 1994; Roberds et al., 1996; Thornton, 2002). Nevertheless, regardless of whether the EH holds, the empirical literature provides evidence that these rates co-move in the long run. [1]

The objective of the present study is to gain some insights into the dynamic relationship between the FF and the TB. Rather than hypothesizing and testing the implications of a particular hypothesis or theory of interest rate behavior, as is frequently done in economics and finance, the approach taken in this paper is agnostic. We investigate the dynamic relationship using a model that relies solely on the fact that the interest rates examined comove in the long run and, therefore, cointegrate. Cointegration and equilibrium correction modeling techniques are a natural way to investigate the relationship between interest rates. The forces that generate a long-run equilibrium relationship between rates at different maturities imply mean reversion of the spread and the existence of an ECM that characterizes the dynamic relationship between the rates. As noted by Granger and Swanson p. 543 (1996), while cointegration is "just a property" of the data, the ECM is a potential data generating mechanism. Consequently, identifying a stable ECM between the FF and the 3-month TB is an important step toward understanding the dynamic relationship between these two interest rates.

Our modeling strategy takes into account the existence of a recent strand of the empirical literature—discussed in detail below—suggesting that interest rate dyna-

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[1] Note that, while cointegration between interest rates at different maturities is necessary for the EH to hold, this requirement is by no means sufficient. For example, Miron (1991) points out that cointegration between rates at different maturities is consistent with several alternative theories of interest rate behavior, including the EH.

mics may be characterized by strong asymmetries and nonlinearities due to factors such as nonzero or asymmetric transactions costs, infrequent trading, or other factors that affect the adjustment of interest rates toward their long-run equilibrium. To date, however, researchers have used dynamic models that allow for either asymmetry or nonlinearity, but not both. We extend the modeling procedures developed in this literature by investigating the dynamic relationship between the FF and the TB using an ECM that allows for *both* asymmetries and nonlinearities in a fairly comprehensive fashion. Our model outperforms several alternative econometric models previously employed in this context.

Using daily data on the effective FF and the 3-month TB for the sample period from 1974 to 1999, we find a long-run relationship between the FF and the TB that is stable over the sample period and across monetary policy regimes. We also find a general nonlinear, asymmetric vector ECM that is temporally stable and significantly outperforms the alternative of a linear ECM.

Our most unexpected result is that the adjustment toward the long-run equilibrium largely occurs through movements in the FF rather than the TB. We note that, prima facie, this finding appears to be inconsistent with the conventional EH/monetary policy view that the FF "anchors" the short end of the US money market. We discuss circumstances that could reconcile the conventional EH/monetary policy interpretation with our results. We undertake several robustness checks designed to investigate these alternative explanations. We conclude that the relationship between the FF and the TB appears to be considerably more complex than simple models of monetary policy or the EH suggest.

The remainder of the paper is set out as follows. Section 2 provides an overview of the theoretical considerations and the empirical literature used to motivate our modeling approach. Section 3 describes the data set. Section 4 contains the empirical results from employing conventional unit root and cointegration tests, executing parameter constancy and linearity tests, and estimating asymmetric linear and nonlinear ECMs. Section 5 reports the results of several robustness checks. The final section briefly summarizes our results and presents the conclusions. The Appendix A provides technical details of how parameter constancy and linearity tests were performed.

2. Modeling the relationship between the FF and the TB

2.1. The expectations hypothesis of the term structure

A number of researchers (e.g., Engle and Granger, 1987; Stock and Watson, 1988; Campbell and Shiller, 1991; Hall et al., 1992) have noted that cointegration and equilibrium correction techniques are natural ways to test the implications of the EH and, more generally, to model the term structure of interest rates. Although it is not the goal of the present study to test the EH per se, it is instructive to show how the approach taken in this paper can be motivated by the EH.

Let $i_{k,t}$ and $f_{k,t}$ be the yield to maturity of a k-period pure discount bond and the forward rate, defined as the contract rate of a one-period pure discount bond bought

at time t that matures k periods ahead. Using the conventional Fisher–Hicks recursive formulae, the relationship linking $i_{k,t}$ and $f_{k,t}$ may be described as follows (e.g., Hall et al., 1992; Campbell et al., 1997):

$$i_{k,t} = \frac{1}{k}\left(\sum_{j=1}^{k} f_{j,t}\right) \quad \text{for } k = 1, 2, 3, \ldots \tag{1}$$

Under the conventional assumption that the relationship between forward rates and expected rates may be characterized as $f_{j,t} = E_t(i_{k,t+j-1}) + \phi_{j,t}$ where $\phi_{j,t}$ is a risk premium required by investors for risk considerations and preferences for liquidity, treating the premia components as time-invariant (as is assumed in the most common and weakest form of the EH), Eq. (1) can be rewritten as

$$i_{k,t} - i_{1,t} = \frac{1}{k}\left[\sum_{i=1}^{k-1}\sum_{j=1}^{i} E_t \Delta i_{1,t+j}\right] + \gamma_k, \tag{2}$$

where E is the expectation operator, Δ denotes the first-difference operator; and $\gamma_{k,t} = \frac{1}{k}\sum_{j=1}^{k}\phi_{j,t}$ is set equal to γ_k because of the assumption of constant risk premia. Eq. (2) links yields at different maturities, implying that yields on assets with similar maturities tend to move together. [2] Also, if discount yields are integrated stochastic processes of order one, $I(1)$, as reported by a large empirical literature (starting at least from Engle and Granger, 1987; and Stock and Watson, 1988), the right-hand-side of (2) is stationary. This implies that the left-hand-side of (2) is stationary, i.e., $(i_{k,t} - i_{1,t}) \sim I(0)$. It is also easy to see that the EH implies that in long-run equilibrium the rates differ by only a constant risk premium and the cointegrating vector linking the rates is $[1, -1]$.

   This framework also has implications for the dynamic structure of the ECM. Specifically, since this framework follows from the Fisher–Hicks formulae that explicitly state that the long-term rate is determined by the market's expectation of the short-term rate over the term of the long-term rate, it implies that the short-term rate is determined independent of the long-term rate. This should be particularly true when the short-term rate is the FF, which is thought to be directly controlled by the Fed. The market forms expectations of the funds rate based on its understanding of monetary policy. Changes in monetary policy induce changes in the FF, which in turn

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[2] The EH of the term structure is often motivated by arbitrage considerations (see, inter alia, Hall et al., 1992) on the ground that departures from the equilibrium relationship between the short-term rate and the long-term rate may imply the possibility of making a riskless profit. Hence, we sometimes use the term arbitrage in this paper. Nevertheless, it should be noted that our use of the term arbitrage is somewhat loose in that it does not exactly match the definition of arbitrage usually employed in finance. Also, note that the absence of arbitrage opportunities does *not necessarily* imply that the EH should hold. As demonstrated by Longstaff (2000), however, all traditional forms of the EH can be consistent with the absence of arbitrage if markets are incomplete. (In turn, this implies that the validity of the EH is mainly an empirical issue, meaning that the EH cannot be ruled out on a priori theoretical grounds.) Note, however, that this comment is only valid under the assumption of constant risk premia; if risk premia are in fact time-varying, then long-term interest rates may move in the absence of a movement in the short-term rate.

move the TB. The reverse is assumed not to be true, however. That is, changes in the 3-month TB do not induce changes in the funds rate. If the Fed controls the funds rate, then the TB may be expected to bear the burden of adjustment to the long-run equilibrium when there is an unexpected change in monetary policy. [3]

2.2. The econometric framework

Our empirical model is essentially the result of generalizing conventional cointegrated vector autoregressions and vector ECMs to a nonlinear framework. Consider the following pth order vector autoregression, VAR(p):

$$y_t = v + \sum_{i=1}^{p} \Gamma_i y_{t-i} + \varepsilon_t, \tag{3}$$

where the K-dimensional observed time series vector $y_t = [y_{1t}, y_{2t}, \ldots, y_{Kt}]'$; v is the vector of intercepts; the Γ_i's are $K \times K$ matrices of parameters; and $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}]'$ is a K-dimensional vector of Gaussian white noise processes with covariance matrix Σ, $\varepsilon_t \sim NID(0, \Sigma)$. Eq. (3) can be rewritten as

$$\Delta y_t = v + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \tag{4}$$

where $\Lambda_i = -\sum_{j=i+1}^{P} \Gamma_j$ for $i = 1, \ldots, p-1$ are matrices of parameters, and $\Pi = \sum_{i=1}^{p} \Gamma_i - \mathbf{I}_K$ is the long-run impact matrix. If y_t is first-difference stationary, i.e., $y_t \sim I(1)$, the rank of Π, $r \leqslant K - 1$ is the number of linearly independent cointegrating vectors and $K - r$ is the number of common trends (e.g., Dickey et al., 1991; Johansen, 1988, 1991, 1995). In this framework, $\Pi = \alpha'\beta$ characterizes the long-run equilibrium of the system. The deviation from the long-run equilibrium is measured by the stationary stochastic process $u_t = \alpha'y_t - \beta$ (Engle and Granger, 1987; Granger, 1986).

The present application focuses on a bivariate model comprising the FF and the TB—i.e., $y_t = [FF_t, TB_t]'$. Hence the series for the deviations from the long-run equilibrium we consider is essentially the cointegrating error term, say u_t, obtained from executing the Johansen (1988, 1991) maximum likelihood cointegration procedure in a VAR(p) involving FF_t, TB_t, and an unrestricted constant term.

As briefly mentioned in the introduction, an interesting strand of this literature has developed recently. This literature allows for asymmetric or nonlinear adjustment toward equilibrium in modeling interest rate movements based on the idea that

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[3] An alternative interpretation is that the EH merely implies that the short-term rate and the long-term rate are determined jointly in equilibrium. Strictly speaking, it may not seem immediately obvious how a change in the long-term rate can occur without a change in the short-term rate, given that—by the Fisher–Hicks formulae—the long-term rate is a weighted average of expected short-term rates. However, this may not be the case if the long-term rate were the 10-year (or the 30-year) Treasury yield. It is well known that policymakers are trying to stabilize inflation and that they may look at long-term Treasury yields as indicators of long-run inflation expectations (e.g., Goodfriend, 1993).

interest rates arbitrage relationships may be characterized by complex nonlinear dynamics. For example, Anderson (1997) uses nonlinear ECMs to study yield movements in the US T-bill market and argues that nonlinear equilibrium correction arises because portfolio adjustment is an "on–off" process, which occurs only when disequilibrium in the bill market is large enough to induce investors to incur the transactions costs associated with buying/selling bills. This, together with heterogeneity of transactions costs, implies that the strength of aggregate equilibrium correction depends on both the distribution of costs and the extent of disequilibrium in the market. Anderson uses smooth transition models with the transition to equilibrium characterized by an exponential function to describe an aggregate adjustment process which is stronger the more distant the market is from equilibrium, but is weak when the market is in the neighborhood of equilibrium.

Enders and Granger (1998) investigate the term structure of interest rates from a different perspective. They employ an asymmetric modeling framework. They also develop critical values to test the null hypothesis of a unit root in the spread between a short-term rate and a long-term rate against the alternative hypothesis of stationarity with asymmetric adjustment toward equilibrium. Within a reasonable range of adjustment parameters, the power of the new tests is shown to be greater than that of the corresponding conventional symmetric unit root tests if the true data generating process is asymmetric. Their results suggest that interest rate movements toward long-run equilibrium may be best captured by an asymmetric process. Enders and Granger also find that the speed of adjustment toward equilibrium is faster for negative deviations from equilibrium than for positive deviations. Their study is primarily methodological. Consequently, Enders and Granger make no attempt to provide an economic interpretation for their finding of an asymmetric adjustment process. Nevertheless, their findings suggest that it may be appropriate to consider a model of interest rates that explicitly allows for this possibility.

In our empirical model, initially we considered an ECM that allows for asymmetric adjustment to long-run equilibrium in a fashion similar to Enders and Granger (1998). Specifically, we estimated the following two-equation system of linear ECMs, using full information maximum likelihood (FIML):

$$\Delta \text{FF}_t = c + \lambda^- u_{t-1}^- + \lambda^+ u_{t-1}^+ + \sum_{i=1}^{p} \gamma_{1i} \Delta \text{FF}_{t-i} + \sum_{i=1}^{p} \gamma_{2i} \Delta \text{TB}_{t-i} + \text{innovations},$$

(5)

$$\Delta \text{TB}_t = k + \rho^- u_{t-1}^- + \rho^+ u_{t-1}^+ + \sum_{i=1}^{p} \theta_{1i} \Delta \text{FF}_{t-i} + \sum_{i=1}^{p} \theta_{2i} \Delta \text{TB}_{t-i} + \text{innovations},$$

(6)

where c and k are constant terms; λ^-, λ^+, ρ^-, and ρ^+, are speed-of-adjustment coefficients (equilibrium correction terms); u_{t-1}^- and u_{t-1}^+ denote negative and positive deviations from the long-run equilibrium implied by the long-run relationship be-

tween the FF and the TB. This model allows for asymmetric responses to lagged deviations from long-run equilibrium in each equation. [4]

Estimates of the asymmetric ECM (5) and (6), however, displayed temporal parameter instability and misspecification, as evidenced by remaining nonlinearity in the residuals. Consequently, we estimated a nonlinear ECM that allows for exponential-type nonlinearity in addition to asymmetric adjustment. Allowing for nonlinearity in the ECM was crucial to obtain parameter stability. It appears that the system (5) and (6) failed because it did not capture adequately the nonlinearity that characterizes the dynamic relationship between the FF and TB. Specifically, we estimated a general two-equation nonlinear, asymmetric ECM in the spirit of the smooth transition regressions suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998):

$$\Delta FF_t = v_1 + \zeta_1^- u_{t-1}^- + \zeta_1^+ u_{t-1}^+ + \sum_{i=1}^p \mu_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \mu_{2i}\Delta TB_{t-i}$$
$$+ \left[v_2 + \zeta_2^- u_{t-1}^- + \zeta_2^+ u_{t-1}^+ + \sum_{i=1}^p \pi_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \pi_{2i}\Delta TB_{t-i} \right]$$
$$\times \left\{ 1 - \exp\left[-\chi(u_{t-d} - v_3)^2 \right] \right\} + \text{innovations}, \tag{7}$$

$$\Delta TB_t = \omega_1 + \xi_1^- u_{t-1}^- + \xi_1^+ u_{t-1}^+ + \sum_{i=1}^p \vartheta_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \vartheta_{2i}\Delta TB_{t-i}$$
$$+ \left[\omega_2 + \xi_2^- u_{t-1}^- + \xi_2^+ u_{t-1}^+ + \sum_{i=1}^p \phi_{1i}\Delta FF_{t-i} + \sum_{i=1}^p \phi_{2i}\Delta TB_{t-i} \right]$$
$$\times \left\{ 1 - \exp\left[-\psi(u_{t-d} - \omega_3)^2 \right] \right\} + \text{innovations}, \tag{8}$$

where v_1 and ω_1 are constant terms; ζ_j^-, ζ_j^+, ξ_j^-, and ξ_j^+ for $j = 1, 2$ are speed-of-adjustment coefficients; u_{t-1}^- and u_{t-1}^+ are as defined above in (5) and (6). The transition functions in the system (7) and (8) are exponential functions. The exponential transition function, say $\Phi(\cdot)$ is bounded between zero and unity, $\Phi: \Re \to [0, 1]$, has the properties $\Phi[0] = 0$ and $\lim_{x \to \pm\infty} \Phi[x] = 1$, and is symmetrically inverse-bell shaped around zero. These properties of the exponential function are attractive in the present modeling context because they allow a smooth transition between regimes and faster adjustment of the interest rates examined for larger deviations above and below the equilibrium level. Because the model (7) and (8) allows for different equilibrium correction coefficients depending upon whether the deviation from equilibrium is positive or negative, it explicitly allows for both asymmetries and nonlinearities in a very general, comprehensive fashion. Indeed, the model (7) and (8) nests the linear asymmetric Enders and Granger (1998) model, the

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[4] Formally, $u_t^+ = u_t$ if $u_t \geqslant 0$, $u_t = 0$ otherwise; $u_t^- = u_t - u_t^+$. Hence, this asymmetric ECM allows a varying strength of attraction to equilibrium where the attractor is assumed to be stronger on one side than on the other, so that u_t^+ may have a different coefficient from u_t^- (see also Granger and Lee, 1989).

symmetric exponential smooth transition model employed by Anderson (1997), and a conventional linear symmetric ECM. The absence of asymmetries and nonlinearities in the dynamics of the interest rates under examination would be exhibited by very similar estimates of the equilibrium correction terms responding to negative and positive deviations from equilibrium (no asymmetry) and/or by the lack of statistical significance of the speed-of-adjustment parameters associated with the nonlinear transition function (no nonlinearity). When this happens, the system (7) and (8) collapses to the conventional linear ECM (4). [5,6]

3. Data

The data are daily observations on the US effective FF and the US 3-month TB over the period from 1 January, 1974 to 31 December, 1999. The effective FF is a weighted average of the rates on federal funds transactions of a group of federal funds brokers who report their transactions daily to the Federal Reserve Bank of New York. Federal funds are deposit balances at Federal Reserve banks that institutions (primarily depositories, e.g. banks and thrifts) lend overnight to each other. These deposit balances are used to satisfy reserve requirements of the Federal Reserve System. Because reserve requirements are binding at the end of the reserve maintenance period, called settlement Wednesday, the funds rate tends to be more volatile on settlement Wednesdays. [7] The FF time series was adjusted in order to eliminate the effect of the increased volatility on settlement days and during three well-known episodes of high volatility that occurred in 1985 and 1986. [8] The TB is

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[5] Essentially our procedure is an example of a bottom-up procedure and consists of starting with a simple but statistically reliable ECM by imposing linearity and then testing the model against alternatives (see Krolzig, 1997).

[6] The class of nonlinear models is infinite. We have chosen to concentrate on the STR formulation primarily because of the above stated properties, its relative simplicity, and the large amount of previous research on the estimation of STR models. Note that Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998) also suggest the logistic function as a plausible transition function for some applications, resulting in a logistic STR or LSTR model. The LSTR model, however, implies asymmetric behavior of the interest rates in question according to whether they are above or below their equilibrium level. Since this behavior is already captured by the differences in the equilibrium correction terms for negative and positive deviations, we expect the LSTR formulation to be somewhat less appropriate in the present context. In addition, the exponential function has a natural interpretation in terms of arbitrage, namely that the adjustment toward equilibrium is faster the larger the deviation from equilibrium. Nevertheless, below, we test for nonlinearities arising from LSTR formulations as a test of specification of our estimated models.

[7] Since February 1984 the reserve maintenance period has been two weeks for all institutions. Before 1984 it was one week for most large institutions. For a more detailed discussion of the Federal Reserve's reserve requirements and the microstructure of the federal funds market, see, for example, Taylor (2001). For comprehensive descriptions of the institutional aspects of the FF market, see Stigum (1990) and Furfine (1999).

[8] Precisely, the adjusted time series for the FF is the ordinary least squares residual from the regression of the FF on a dummy which is equal to unity for the three data points corresponding to the outliers mentioned above and zero for all remaining data points, and a dummy variable that equals zero on nonsettlement days and unity on settlement days.

the daily market closing rate in the secondary market. Both the FF and the TB data are expressed as bond equivalent yields on a 365-day basis. [9]

In our sensitivity analysis, discussed in Section 5, we also use daily time series for the 3-month rate on certificates of deposit (CD rate) for the period from 1 January, 1974 to 31 December, 1999, and for the FF target (FF^{target}) for the period from 22 November, 1989 to 31 December, 1999. [10] We also use monthly data on the federal funds and TBs for the period from January 1974 to December 1999. The monthly time series data are expressed as end-of-month daily data. All of these rates are expressed as bond equivalent yields on a 365-day basis.

The time series for FF, TB and CD were taken from the Federal Reserve Bank of St. Louis database, federal reserve economic data. The main sample period—1 January, 1974–31 December, 1999—covers 25 years, a period that should be sufficiently long to capture some of the main features of the unknown stochastic process governing the relationship between the FF and the TB. Also, the number of observations, $T = 6784$, is sufficiently large to be fairly confident of the estimation results.

4. Empirical analysis

4.1. Preliminary statistics and cointegration analysis

Table 1 presents summary statistics as well as the results of unit root tests for FF and TB. [11] In contrast with the usual view that the term structure of interest rates, which suggests that the yield curve is typically upward sloping, FF averaged about 52 basis points more than TB. One possible explanation is that the markets for federal funds and T-bills are at least partially segmented, i.e., the two rates in question belong to different segments of the US money market (see Campbell et al., 1997). This view, however, is diametrically opposite to the view that the FF affects all other interest rates at the short end of the term structure because of its key role in the implementation of monetary policy. Another possibility is that Treasury rates are free of default risk, while the funds rate is not. It would appear, however, that only a small fraction of the difference in means can be attributed to the default-risk-free character of the TB. A more plausible explanation is that the interest on T-bills is exempt from some local and state taxes. This effectively lowers the nominal TB on average relative to the FF. Given these tax differences, we think that it seems

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[9] The use of bond equivalent yields was suggested by one of the referees. The previous version of the paper used the more commonly reported discount quotes. With the exception of the cointegration parameter on the TB, which was about −1.2 when discount quotes were used, all of the results were qualitatively identical to those reported here. The results using discount quotes will be provided upon request.

[10] The funds rate target can be obtained, for example, from Thornton and Wheelock (2000, Table B1).

[11] In all statistical tests executed in this and subsequent sections, we use a 5% nominal significance level, unless otherwise specified.

Table 1
Preliminary data analysis
Descriptive statistics

	Minimum	Maximum	Mean	Stand. deviation	Skewness	Kurtosis
FF	2.1968	22.3192	7.6040	3.4019	1.3024	1.9135
TB	2.6638	18.1651	7.0766	2.9062	1.2526	1.5545

ADF unit root tests

FF	TB	ΔFF	ΔTB	Δ^2FF	Δ^2TB
−2.05	−2.27	−44.69	−31.05	−65.05	−57.43

Notes: FF and TB denote the FF and the 3-month TB respectively; Δ is the first-difference operator. Figures reported are annualized; the sample period spans from 1974:1:1 to 1999:12:31, and the number of observations is 6784. For ADF tests, the critical value at the 5% (1%) level of significance is −2.86 (−3.43) to two decimal places (Dickey and Fuller, 1979; MacKinnon, 1991).

reasonable to assume that FF and TB belong to the same segment of the US money market, as a large empirical literature has assumed.

The FF is considerably more variable than the TB. This is not surprising given the relative size of the federals funds and T-bill markets and the nature of the federal funds market. An examination of the third and fourth moments indicates the existence of both excess skewness and kurtosis, suggesting that the underlying distribution of both FF and TB may be nonnormal.

In keeping with a large number of studies of unit root behavior of interest rates, using standard augmented Dickey–Fuller (ADF) test statistics, we were unable to reject the unit root null hypothesis for both FF and TB at conventional nominal levels of significance. Nevertheless, ADF tests on first- and second-differences of the two rates examined indicate that the differenced series are $I(0)$ for each rate, therefore supporting the stylized fact that interest rates are $I(1)$ processes (e.g., Stock and Watson, 1988, 1999).

There is an apparent conflict between conventional economic and finance theory, which often assumes that interest rates are stationary processes, [12] and the mainstream empirical literature on interest rates, which (at least since Engle and Granger, 1987) either assumes or finds that interest rates are nonstationary processes. We follow the empirical practice because very persistent series with a root at least very close (if not equal) to unity are better approximated by $I(1)$ processes than by stationary ones (see, for example, Stock, 1997 and Lanne, 1999, 2000 and the references therein).

The Monte Carlo results provided by Balke and Fomby (1997) indicate that estimation of the long-run equilibrium using the Johansen (1988, 1991) procedure when the true adjustment toward equilibrium is nonlinear does not yield misleading results in terms of significant loss of power or size distortion. Hence, we use the Johansen maximum likelihood cointegration procedure to test for cointegration between FF

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[12] For example, see the vast finance literature assuming a Vasicek (1977) model of interest rates, which is simply a mean-reverting process representable as an Ornstein–Uhlenbeck equation.

Table 2
Johansen maximum likelihood cointegration procedure: Panel A: and Panel B:

H_0	H_1	$\lambda_{\max}$	CV	H_0	H_1	λ_{trace}	CV
Panel A: Cointegration LR tests based on maximum eigenvalue of the stochastic matrix (λ_{max}) and on trace eigenvalue of the stochastic matrix (λ_{trace})							
$r = 0$	$r = 1$	907.6 (907.4)	14.1	$r = 0$	$r \geqslant 1$	909.8 (909.5)	15.4
$r \leqslant 1$	$r = 2$	3.002 (3.000)	3.8	$r \leqslant 1$	$r = 2$	3.002 (3.000)	3.8
Panel B: Estimated cointegrating vector							

FF	TB	Constant
1.0000	−1.1501	0.5175
[–]	[0.0094]	[0.0942]

Notes: These results were obtained from testing for cointegration in a VAR(5) comprised of FF, TB and an unrestricted constant term. In Panel A, H_0 and H_1 denote the null hypothesis and the alternative hypothesis respectively; figures in parentheses are test statistics adjusted for degrees of freedom (Reimers, 1992); r denotes the number of cointegrating vectors, and CV is the 95% critical value (see Osterwald-Lenum, 1992; Johansen, 1995). In Panel B, figures in square brackets are estimated standard errors.

and TB and to estimate the equilibrium error. Panel A of Table 2 reports the maximum eigenvalue and trace statistics from the Johansen procedure in a VAR(5) composed of FF, TB and an unrestricted constant term. The results with and without Reimers's (1992) adjustment for degrees of freedom indicate that there is a unique long-run relationship between the FF and the TB.

Having established the rank of the long-run impact matrix Π, we re-executed the Johansen cointegration procedure under the restriction that the rank of Π equals unity and the nonbinding normalization that the coefficient on FF_t is unity. Panel B of Table 2 then reports the estimated cointegrating vector. The estimated cointegrating relationship appears remarkably stable over time (across different monetary policy regimes), as shown visually by the recursive estimate of the unrestricted parameter on TB, plotted in Fig. 1, along with plus/minus twice the corresponding standard errors. The estimated coefficient on TB is correctly signed, but the hypothesis that it is equal to minus unity (the theoretical value implied by the EH) is easily rejected at conventional significance levels. The fact that the cointegrating relationship is consistent with a cointegrating parameter of about −1.15 rather than −1.0 might be seen as further corroborating the idea that it is not *exactly* the EH of the term structure that binds the two rates. [13] The rejection of the hypothesis that the cointegrating parameter in term structure relationships is −1.0 is quite common in studies of the EH using US data (e.g., Bekaert et al., 1997) as well as studies of interest rate

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[13] The relevant literature suggests a number of reasons capable of explaining the violation of the unity restriction in term-structure cointegrating regressions of this kind. One possibility is, for example, that the violation is due to the existence of time-varying risk premia such that the long-term interest rate is in fact related to the level of the short-term interest rate (see Fama, 1984).
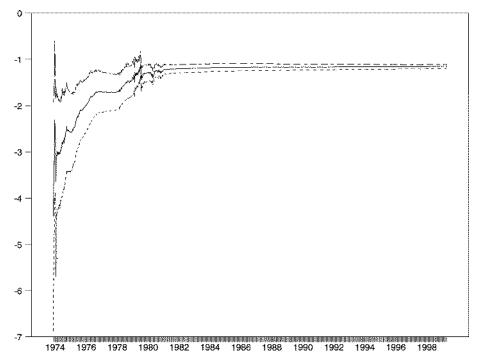
Fig. 1. Recursive estimate of the cointegrating parameter: solid line—cointegrating parameter (*b*), broken lines—$b \pm 2.0$ times standard error.

convergence in the context of the European Monetary System (e.g. Siklos and Wohar, 1996, 1997; Granger and Siklos, 1999, and the references therein). Nevertheless, the evidence of the existence of a long-run relationship between FF and TB is very strong. The cointegrating residual or estimated equilibrium error implied by this long-run relationship, $\hat{u}_t = \mathrm{FF}_t - \hat{\alpha} - \hat{\beta}\mathrm{TB}_t$, essentially measures the deviation from the cointegrating equilibrium relationship, so that when $\hat{u}_t > 0$ ($\hat{u}_t < 0$) the FF rate is too high (low) relative to the equilibrium relationship, while the TB rate is too low (high).

4.2. Asymmetric equilibrium correction models

Preliminary to considering a nonlinear ECM, we estimated the bivariate asymmetric ECM (5) and (6). This model explicitly allows for an asymmetric response of interest rates to negative and positive deviations from long-run equilibrium in a fashion similar to Enders and Granger (1998). We estimated the model by FIML assuming a lag length of five, as suggested by standard information criteria, and employed the conventional general-to-specific procedure to obtain parsimonious ECMs for each of $\Delta\mathrm{FF}_t$ and $\Delta\mathrm{TB}_t$ (see Hendry et al., 1984). The resulting models, presented in Table 3, appear to be adequate in terms of approximately white noise residuals, although the adjusted coefficient of determination is not particularly high

Table 3
Linear asymmetric ECMs

Dep. variable: ΔFF_t		Dep. variable: ΔTB_t	
Panel A: Estimated parsimonious models			
c	–	k	–
λ^-	−0.258	ρ^-	6.3×10^{-3}
	(0.008)		(1.4×10^{-3})
λ^+	−0.059	ρ^+	1.2×10^{-3}
	(0.018)		(0.6×10^{-3})
γ_{11}	−0.318	θ_{11}	0.020
	(0.014)		(3.5×10^{-3})
γ_{12}	−0.204	θ_{12}	0.021
	(0.014)		(3.4×10^{-3})
γ_{13}	−0.189	θ_{13}	9.4×10^{-3}
	(0.013)		(3.1×10^{-3})
γ_{14}	−0.162	θ_{14}	–
	(0.013)		
γ_{15}	0.185	θ_{15}	0.011
	(0.012)		(2.9×10^{-3})
γ_{21}	0.230	θ_{21}	0.103
	(0.055)		(0.012)
γ_{22}	0.186	θ_{22}	–
	(0.044)		
γ_{23}	0.096	θ_{23}	−0.044
	(0.045)		(0.012)
γ_{24}	0.102	θ_{24}	–
	(0.043)		
γ_{25}	−0.104	θ_{25}	0.059
	(0.041)		(0.011)
Panel B: Diagnostics			
$\overline{R}^2$	0.326	0.030	
$J - ECM$	$\{5.2 \times 10^{-62}\}$	$\{6.2 \times 10^{-8}\}$	
Asymmetry	$\{6.1 \times 10^{-15}\}$	$\{3.8 \times 10^{-10}\}$	
DW	1.968	2.001	
LB(20)	$\{0.584\}$	$\{0.439\}$	
LR	$\{0.732\}$	$\{0.420\}$	
s	0.207	0.016	

Notes: In Panel A, the model estimated is given by the system of equilibrium correction equations (5) and (6). The parsimonious models were obtained by estimating the system by FIML and applying a general to specific procedure; figures in parentheses denote estimated standard errors. In Panel B, $\overline{R}^2$ is the adjusted coefficient of determination; $J - ECM$ is a test of joint significance of the two equilibrium correction coefficients in each equation (i.e., $H_0 : \lambda^- = \lambda^+ = 0$ and $H_0 : \rho^- = \rho^+ = 0$), and is distributed as $\chi^2(2)$ under the null; Asymmetry is a test of the null hypothesis of no asymmetric equilibrium correction; DW is the Durbin–Watson statistic; LB(20) is the Ljung–Box test for residual autocorrelation up to order 20, and is distributed as $\chi^2(20)$ under the null; LR is a likelihood ratio test for the validity of the restrictions imposed in the final estimation; s is the RV. Figures in braces denote *p*-values.

for the equation for ΔFF and is very low for the equation for ΔTB. Both λ^- and λ^+ are strongly individually significant in the equation for ΔFF. In contrast, only ρ^- is strongly individually significant in the equation for ΔTB. Nevertheless, joint tests of

statistical significance of the equilibrium correction terms ($J - \text{ECM}$, testing $H_0 : \lambda^- = \lambda^+ = 0$ and $H_0 : \rho^- = \rho^+ = 0$ in Eqs. (5) and (6), respectively) indicate strong rejection of the null hypothesis of no equilibrium correction for both equations. Like Enders and Granger (1998), we find that the speed of adjustment is faster for negative than for positive deviations from equilibrium. The difference in the estimates of the equilibrium correction terms for positive and negative deviations from equilibrium is quite large for both equations, suggesting that the allowance for asymmetric adjustment is very important in this context for each equilibrium correction equation. [14] Moreover, the estimates suggest that most of the adjustment toward long-run equilibrium occurs through the FF, rather than the TB.

4.3. Parameter constancy and linearity tests

In order to evaluate the adequacy of the asymmetric ECMs reported in Table 3, we initially executed some tests of parameter constancy against the alternative of smoothly changing parameters (Lin and Teräsvirta, 1994; see Appendix A). The tests results, reported in Panel A of Table 4, indicate a strong rejection of the hypothesis of parameter constancy against the alternative hypothesis of smoothly-changing parameters, suggesting that nonlinear equilibrium correction may be a prerequisite for parameter stability.

Some evidence of general nonlinearity is provided by the RESET test statistics (Ramsey, 1969). The RESET test is a fairly general misspecification test. Under the RESET test statistic, the alternative model involves a higher-order polynomial to represent a different functional form. Under the null hypothesis, the statistic is distributed as $\chi^2(g)$ with g equal to the number of higher-order terms in the alternative model. [15] Panel B of Table 4 reports the RESET test statistics applied to the model in Table 3 constructed using squared and cubed fitted values. The results indicate a strong rejection of the null hypothesis of linearity (no misspecification) with p-values of virtually zero.

In addition, we performed tests of linearity against the alternative of smooth transition nonlinearity and followed the Teräsvirta (1994, 1998) decision rule to select the most adequate transition function for modeling nonlinearity in the present context (see Appendix A). As shown by the results in Panel C of Table 3, the general linearity test F_L easily rejects the null hypothesis of linearity, especially for small delay parameters. The strongest rejection occurs when $d = 1$ for both equations, suggesting a fast response to disequilibria. Employing the Teräsvirta rule to discriminate between

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[14] When the equations are estimated imposing the restrictions $\lambda^- = \lambda^+ = \lambda$ and $\rho^- = \rho^+ = \rho$, the estimate of λ is −0.1023 (0.0080) and the estimate of ρ is 0.0045 (0.0019); figures in parentheses are standard errors. Nevertheless, testing the null hypothesis of no asymmetry in the equilibrium correction, namely $H_0 : \lambda^- = \lambda^+ = \lambda$ and $H_0 : \rho^- = \rho^+ = \rho$ in Eqs. (5) and (6), respectively, yielded rejections of the null with p-values of virtually zero, as reported in Panel B of Table 3.

[15] In constructing the tests we use the F-statistic form. It is well known that in finite samples the actual test size of the F approximation may be closer to the nominal significance level than the actual size of the χ^2 approximation (see Teräsvirta, 1994).

Table 4
Parameter constancy, misspecification, and linearity tests

	Dep. variable: ΔFF_t	Dep. variable: ΔTB_t
(A) Lin–Teräsvirta parameter constancy tests		
LM_{C3}	5.2×10^{-12}	1.4×10^{-5}
LM_{C2}	5.5×10^{-75}	1.2×10^{-8}
LM_{C1}	3.4×10^{-53}	6.0×10^{-2}
(B) RESET test statistics		
4.5×10^{-22}	7.2×10^{-40}	
(C) Teräsvirta linearity tests		
General linearity test F_L $(p = 5)$		
$d = 1$	5.2×10^{-41}	5.7×10^{-19}
$d = 2$	4.8×10^{-22}	3.4×10^{-10}
$d = 3$	5.5×10^{-24}	7.5×10^{-7}
$d = 4$	1.6×10^{-30}	2.4×10^{-5}
$d = 5$	7.4×10^{-6}	6.0×10^{-3}
$d = 6$	5.3×10^{-21}	7.1×10^{-3}
$d = 7$	6.2×10^{-30}	8.3×10^{-2}
$d = 8$	2.0×10^{-12}	9.2×10^{-3}
$d = 9$	1.6×10^{-5}	5.2×10^{-2}
$d = 10$	4.9×10^{-2}	1.3×10^{-1}
Teräsvirta rule $(p = 5; d = 1)$		
F_L	5.2×10^{-41}	5.7×10^{-19}
F_3	7.2×10^{-8}	3.2×10^{-4}
F_2	5.3×10^{-38}	9.1×10^{-14}
F_1	4.2×10^{-23}	6.3×10^{-3}

Notes: All test statistics recorded in this table were executed on the parsimonious linear ECMs reported in Table 3. In Panel A, figures are *p*-values from Lin–Teräsvirta test statistics of the null hypothesis of parameter constancy against the alternative hypothesis of smooth-transition-type nonconstancy (see Appendix A for details). In Panel B, figures are *p*-values from RESET test statistics. In Panel C, figures are *p*-values from Teräsvirta test statistics of the null hypothesis of linearity against the alternative hypothesis of smooth-transition-type nonlinearity using the cointegrating residuals $\hat{u}_{t-d}$ as the transition variable (see Appendix A for details).

ESTR and LSTR formulations led us to conclude that an ESTR is the most adequate parametric STR formulation (given that F_2 yields the lowest *p*-value). This finding is consistent with our priors, discussed in Section 2, and with Anderson (1997) arguments that interest rate arbitrage is an ''on–off'' process that occurs only when disequilibrium in the interest rate market is large enough to induce investors to incur heterogeneous transactions costs. Thus, the strength of aggregate equilibrium correction depends on the extent of the disequilibrium.

4.4. Nonlinear asymmetric equilibrium correction models

Given the results from the linearity tests, the bivariate system of nonlinear ECM for ΔFF and ΔTB, (7) and (8), was estimated by nonlinear FIML (NFIML) with $\hat{u}_{t-1}$

taken as the equilibrium correction term and the transition variable. In estimation, we followed the recommendation of Granger and Teräsvirta (1993) and Teräsvirta (1998) of standardizing the transition parameter by dividing it by the sample variance of the transition variable, $\hat{\sigma}_u^2$, and using a starting value of unity for the estimation algorithm. We then applied a conventional general-to-specific procedure in order to reach parsimonious empirical models. The parsimonious models were obtained by ''testing down'' the general system of ECMs with a lag length of five, imposing exclusion restrictions on the coefficient with the lowest (in absolute size) insignificant *t*-ratio and re-estimating the system sequentially. We repeated the estimation procedure several times using different alternative sequences and different sets of starting values for the parameters in order to ensure that the results were robust to the specification search rule and that a global optimum was achieved.

The results, reported in Panel A of Table 5, indicate that the dynamics of the interest rates examined is highly nonlinear. The estimated standardized transition parameter appears to be strongly significantly different from zero, in each equation, both on the basis of the individual *t*-ratios as well as on the basis of the strong rejection of the Skalin's (1998) parametric bootstrap likelihood ratio test (see SLR in Panel B). Since these tests may be construed as tests of nonlinear equilibrium correction, the results indicate strong evidence of nonlinear equilibrium correction for each interest rate.

The estimated speed-of-adjustment parameters imply well-defined and fairly fast transition functions. This is shown in Fig. 2, which displays the plots of the estimated transition functions for ΔFF and ΔTB against $\hat{u}_{t-k} - \hat{v}_3$ and $\hat{u}_{t-k} - \hat{\omega}_3$, respectively. The limiting case of $\Phi(\cdot) = 1$ is attained in each case. Nevertheless, as the estimated transition functions make clear, the speed of adjustment for ΔFF is larger than for ΔTB. A 20% absolute deviation from equilibrium implies a 75% movement in the transition function of the ΔTB equation, whereas the estimated transition function for ΔFF would be at its limiting value of unity for the same absolute deviation. The difference in the estimated nonlinear equilibrium corrections is further highlighted by Fig. 3, which shows the nonlinear equilibrium corrections implied by our estimates for positive and negative deviations from equilibrium and for both equations in the system (7) and (8). Exponential-type nonlinearity has a small role in the equation for ΔFF, but no role in the equation for ΔTB, given that $\xi_2^- = \xi_2^+ = 0$. The role of asymmetries in the dynamics toward equilibrium, however, is very strong in both equations. Fig. 3 also confirms that the adjustment toward equilibrium occurs largely through movements in the FF rather than the TB.

A battery of diagnostic tests are reported in Panel B of Table 5. As indicated by the joint tests of statistical significance of the equilibrium correction terms that are not restricted to zero in the final estimation, the hypothesis of significant equilibrium correction ($J - ECM$, testing $H_0 : \zeta_2^- = \zeta_2^+ = 0$ and $H_0 : \xi_1^- = \xi_1^+ = 0$ in Eqs. (7) and (8), respectively) is strongly statistically significant for each equation; however, the equilibrium correction terms associated with the nonlinear transition function enter significantly only in the equation for ΔFF (indeed $\xi_2^- = \xi_2^+ = 0$ is imposed in the final estimation). A test of the null hypothesis of no asymmetry indicated strong rejection of the null for both equations (see Asymmetry in Panel B of Table 5). The residual

Table 5
Nonlinear asymmetric ECMs

Dep. variable: ΔFF_t		Dep. variable: ΔTB_t	
(A) Estimated parsimonious models			
v_1	–	ω_1	–
ζ_1^+	–	ξ_1^+	3.9×10^{-3}
			(2.1×10^{-3})
μ_{11}	-0.312	ϑ_{11}	0.103
	(0.026)		(0.020)
μ_{12}	-0.203	ϑ_{12}	–
	(0.024)		
μ_{13}	–	ϑ_{13}	–
μ_{14}	-0.148	ϑ_{14}	–
	(0.016)		
μ_{15}	0.181	ϑ_{15}	–
	(0.020)		
μ_{21}	0.209	ϑ_{21}	0.017
	(0.081)		(5.8×10^{-3})
μ_{22}	0.214	ϑ_{22}	0.018
	(0.101)		(5.1×10^{-3})
μ_{23}	–	ϑ_{23}	–
μ_{24}	–	ϑ_{24}	–
μ_{25}	–	ϑ_{25}	0.011
			(3.8×10^{-3})
v_2	-0.053	ω_2	–
	(0.009)		
ζ_2^-	-0.253	ξ_2^-	–
	(0.024)		
ζ_2^+	-0.034	ξ_2^+	–
	(0.013)		
π_{11}	0.101	ϕ_{11}	–
	(0.027)		
π_{12}	0.065	ϕ_{12}	–
	(0.025)		
π_{13}	-0.214	ϕ_{13}	–
	(0.018)		
π_{14}	–	ϕ_{14}	–
π_{15}	–	ϕ_{15}	0.189
			(0.071)
π_{21}	–	ϕ_{21}	–
π_{22}	–	ϕ_{22}	–
π_{23}	–	ϕ_{23}	0.068
			(0.013)
π_{24}	–	ϕ_{24}	–
π_{25}	–	ϕ_{25}	–
χ	0.910	ψ	0.305
	(0.223)		(0.091)
v_3	–	ω_3	–
(B) Diagnostics			
$\overline{R}^2$	0.396	0.035	
$J - ECM$	$\{3.4 \times 10^{-28}\}$	$\{7.2 \times 10^{-3}\}$	
Asymmetry	$\{4.5 \times 10^{-19}\}$	$\{1.7 \times 10^{-10}\}$	

Table 5 (continued)

Dep. variable: ΔFF_t		Dep. variable: ΔTB_t
$Q(1)$	{0.582}	{0.402}
$Q(20)$	{0.942}	{0.375}
LR	{0.680}	{0.349}
$NLES_{MAX}$	{0.572}	{0.421}
$NLLS_{MAX}$	{0.303}	{0.954}
SLR	{4.3×10^{-26}}	{2.6×10^{-3}}
LM_{C3}	{0.490}	{0.420}
LM_{C2}	{0.504}	{0.712}
LM_{C1}	{0.589}	{0.485}
s	0.173	0.012
(C) Relative goodness of fit: Nonlinear versus linear ECM		
$\overline{R}^2$ ratio	1.21	1.17
RV ratio	0.83	0.75
AIC ratio	1.12	1.18
SIC ratio	1.14	1.19

Notes: In Panel A, the model estimated is given by the system of nonlinear equilibrium correction Eqs. (7) and (8). The parsimonious models were obtained by estimating the system of nonlinear equilibrium correction equations by NFIML and applying a general to specific procedure; figures in parentheses denote estimated standard errors. In Panel B, $\overline{R}^2$ is the adjusted coefficient of determination; $J - ECM$ is a test of joint significance of the equilibrium correction coefficients not restricted to zero in the final estimation in each equation (i.e., $H_0 : \zeta_2^- = \zeta_2^+ = 0$ and $H_0 : \xi_1^- = \xi_1^+ = 0$), and is distributed as $\chi^2(2)$ under the null; Asymmetry is a test of the null hypothesis of no asymmetric equilibrium correction; $Q(1)$ and $Q(20)$ are Lagrange multiplier test statistics for first-order and up to twentieth-order serial correlation in the residuals respectively (Eitrheim and Teräsvirta, 1996); LR is a likelihood ratio test for the validity of the restrictions imposed in the final estimation. $NLES_{MAX}$ is the maximal Lagrange multiplier test statistic for no remaining exponential-smooth-transition-type nonlinearity with delay in the range from 2 to 10; $NLLS_{MAX}$ is the maximal Lagrange multiplier test statistic for no remaining logistic-smooth-transition-type nonlinearity with delay in the range from 1 to 12 (Eitrheim and Teräsvirta, 1996). SLR is a parametric bootstrap likelihood ratio test of the null hypothesis that the transition parameters (χ and ψ) are zero (Skalin, 1998). LM_{C3}, LM_{C2} and LM_{C1} are Lin–Teräsvirta test statistics of the null hypothesis of parameter constancy; s is the RV. Figures in braces denote p-values. In Panel C, the $\overline{R}^2$ ratio, the RV ratio, the AIC and the SIC ratios are the ratios of the $\overline{R}^2$, the RV, the AIC and the SIC from each of the two estimated nonlinear equations reported in Panel A to the corresponding goodness of fit measures obtained for the alternative linear models reported in Table 3.

diagnostics are satisfactory in each case. Eitrheim and Teräsvirta's (1996) tests for residual serial correlation were not found to be statistically significant at conventional nominal levels of significance. We also tested for the stability of the model by constructing Lin and Teräsvirta (1994) tests for each nonlinear ECM. The results suggest no structural break, with p-values reasonably larger than the conventional 5 percent, again indicating that nonlinear equilibrium correction is important for parameter stability in this context. For each of the estimated nonlinear models, the null hypothesis of no remaining nonlinearity for values of d ranging from 2 to 20 could not be rejected on the basis of Lagrange multiplier tests (in Table 5 we report only the maximal value of the LM statistic testing for remaining ESTR nonlinearity, $NLES_{MAX}$). Neither could we reject the hypothesis of no remaining nonlinearity of the LSTR variety with values of the delay parameter in the range from 1 to 20
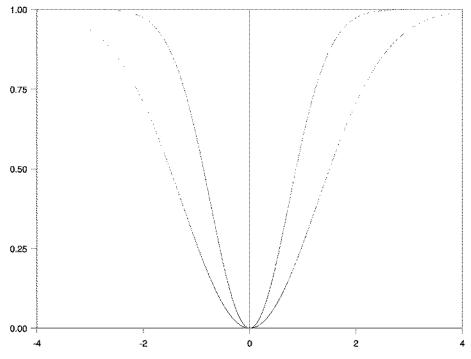
Fig. 2. Estimated transition functions: FF (inner line) and TB (outer line) equations.

(NLLS$_{MAX}$ in Table 5). This procedure therefore confirmed the validity of the choice of $d = 1$.

Although the adjusted coefficient of determination is not particularly high for the equation for ΔFF and is very low for the equation for ΔTB, there is an improvement in terms of goodness of fit relative to the asymmetric linear ECMs reported in Table 3. In order to explicitly compare the goodness of fit of the nonlinear equations (7) and (8) to the linear equations (5) and (6), we calculated the ratio of the $\overline{R}^2$, the residual variance (RV), the Akaike information criterion (AIC), and the Schwartz information criterion (SIC) from each of the estimated nonlinear equations (7) and (8) (reported in Panel A of Table 5) to the corresponding measure for the alternative models in Table 3. The results, reported in Panel C of Table 5, show that for each interest rate the estimated nonlinear asymmetric ECM largely outperforms the best alternative linear model, leading to a substantial reduction in the RV (17% and 25% for the funds rate and TB, respectively).

Overall, the nonlinear estimation results indicate the presence of complex nonlinear dynamics in the relationship between the FF and the TB, with adjustment toward the long-run equilibrium occurring at a speed which depends both upon the sign of the deviation from the equilibrium and on the absolute size of the deviation itself. The estimated models are in every case statistically well determined, provide good fits to the data and pass a battery of diagnostic tests.
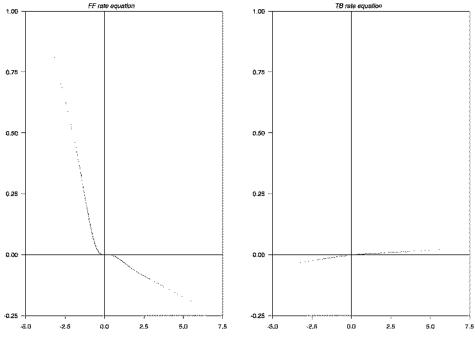
Fig. 3. Nonlinear equilibrium correction versus deviation from equilibrium.

4.5. Interpreting the empirical results

Our empirical results suggest the existence of a unique cointegrating equation characterizing the long-run equilibrium of the FF and the TB. The evidence suggests significant nonlinearities and asymmetry in the adjustment toward the long-run equilibrium between the FF and the TB. Specifically, the speed of adjustment toward the long-run equilibrium between these rates is a function of both the sign and the magnitude of the deviation from equilibrium. These findings accord with an emerging literature on modeling the term structure of interest rates (e.g., Anderson, 1997; Enders and Granger, 1998), suggesting that significant nonlinearities and asymmetries are empirical regularities in the dynamic adjustment of interest rates. [16]

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[16] While the finding of exponential-type nonlinearity may be naturally interpreted in terms of interest rate arbitrage under transactions costs, the finding of asymmetric equilibrium correction (stronger for negative deviations from equilibrium) is more difficult to rationalize. One possibility is that the Fed is more aggressive in raising the funds rate when it is below the structure that links it to the TB (a negative deviation from long-run equilibrium reflects a situation where the funds rate is low relative to the TB) than when it is above the TB. Taking this argument forward, the asymmetry suggests that the Fed may be more prone to raise the funds rate than to lower it. This possibility may stem from the observation that price stability is the Fed's primary policy objective, so that the Fed may be more concerned when the funds rate is relatively low than when it is relatively high. However, we understand that we are examining day-to-day variations in the FF, which may partly be explained by factors that are not related to monetary policy. Therefore, we suggest that this argument be taken merely as a conjecture.

Not all of our results were expected. For example, we were surprised by the re-
markable stability of the long-run relationship between the FF and the TB in light
of the marked changes in Federal Reserve operating procedures that occurred during
the sample period. [17] The Fed was explicitly targeting the funds rate from 1974 to
October 1979. In October 1979, however, the Fed switched to a nonborrowed re-
serves operating procedure in an attempt to reduce inflation by reducing the growth
of the M1 monetary aggregate (Meulendyke, 1998). In October 1982 the Fed
switched to a borrowed reserves operating procedure. Exactly when the Fed switched
from a borrowed reserve operating procedure to an explicit funds rate targeting pro-
cedure is contentious. Thornton (1988) presents evidence that the Fed was explicitly
targeting the funds rate as early as 1984. Meulendyke (1998), however, suggests the
switch was somewhat later, noting that the ''informal move away from borrowing
reserves operating procedure was speeded by the stock market break on 19 October,
1987,'' when the Federal Open Market Committee (FOMC) found that ''a stable re-
lationship between the amount of borrowing and the fund rate did not reemerge''
(Meulendyke, 1998, p. 55). In addition, Hamilton and Jordá (2001) argue that the
Fed was explicitly targeting the FF by late 1989. Consequently, there seems to be
general agreement that the Fed has explicitly targeted the funds rate at least since
the late 1980s.

In any event, if the process governing the funds rate and its relationship with the
TB reflects the policy considerations of the Fed, one might expect to see changes in
the equilibrium relationship between these rates over time. Our results suggest that
the long-run relationship between the FF and the TB was not affected by changes
in the Fed's operating procedure. It should be noted, however, that the simple linear
dynamic model displayed parameter instability. A complex nonlinear ECM was a
prerequisite to achieve parameter stability.

Somewhat more surprising was our finding that the burden of the adjustment to-
ward equilibrium is borne by the FF. If the Fed controls short-term interest rates by
controlling the effective FF, one might expect that the TB would adjust to the funds
rate, rather than the other way around. Of course, this is not necessarily the case. For
example, if the market anticipates changes in the FF, the TB will move in advance of
the funds rate. In this case, the data would give the impression that the funds rate is
adjusting to the TB when, in fact, the TB adjusts to expected changes in the FF. Of
course, this explanation applies only when the market correctly anticipates changes
in the funds rate. When changes in the funds rate are unexpected, the TB would
adjust to the new level of the funds rate.

The possibility that the finding that the funds rate bears the burden of the adjust-
ment process is actually due to the market anticipating changes in the funds rate
would seem to be more likely when the Fed is explicitly targeting the funds rate.
It is well known that the Fed does not adjust its funds rate target immediately in re-
sponse to new information. Rather, target adjustments occur at discrete intervals

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[17] One possibility is that changes in monetary policy operating procedures have induced changes in the
dynamics of the relationship but not in the long-run equilibrium between the funds rate and the TB. Also,
these changes might be reflected in time-varying risk premia over the sample examined.

and by relatively large amounts (e.g., Goodfriend, 1991). Consequently, changes in the level of the funds rate should be less continuous when the Fed is targeting the funds rate than when the funds rate is free to respond to market pressures. Moreover, changes might be easier to predict because the funds rate would be directly linked to the policy objectives of the Fed. In this respect, it is important to note that there have been a number of procedural changes recently that should have significantly improved the market's ability to anticipate changes in the Fed's FF target. Since 1994 the Fed has announced target changes immediately upon making them. Before 1994, target changes were not announced: the market had to infer the Fed's actions by observing open market operations and the FF (e.g., Cook and Hahn, 1989; Rudebusch, 1995; Taylor, 2001). In addition, since 1994, with two exceptions, the Fed has changed the FF target at regular meetings of the FOMC, which are scheduled far in advance. [18] Prior to that, most target changes were made during the inter-meeting period and at the discretion of the Chairman. Also, since October 1989, the Fed has followed the practice of changing the funds rate target by either 25 or 50 basis points, whereas the previous practice involved making target changes of various amounts. [19] These procedural changes should have improved the market's ability to predict the funds rate. However, the fact that our nonlinear asymmetric ECM passes a battery of parameter constancy tests specifically designed for the type of nonlinear model used in this paper (see Panel C of Table 5) indicates that these procedural changes have been *statistically* unimportant for the relationship between the funds rate and the TB in that the equilibrium correction terms are time-invariant over the sample period examined. [20]

There is another reason why the relatively rapid adjustment of the funds rate is not necessarily at odds with the view that the Fed controls the structure of short-term rates; it is directly linked to the Fed's funds rate targeting procedure. If the TB is set more or less equal to the expected path for the funds rate target, the finding that the funds rate bears the burden of the daily adjustment process might merely reflect the fact that the funds rate tends to revert back to the funds rate target when the Fed is explicitly targeting the funds rate. The Fed could play a role in the adjustment process by injecting reserves when the funds rate is above the target and draining reserves when the funds rate is below the target. There is some evidence that the Fed did this during the 1970s (e.g., Cook and Hahn, 1989). Since the early 1980s the Fed has followed the practice of entering the market only once a day, usually before the bulk of federal funds transactions takes place. Consequently, since then the Fed has not actively attempted to offset intra-day deviations of the funds rate from the funds rate target. The Fed can, however, respond to deviations of the funds rate

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[18] The exceptions occurred on 18 April, 1994 and 15 October, 1998.

[19] There was one exception. On 15 October, 1994 the Fed raised the funds rate target by 75 basis points.

[20] In addition to the Lin and Teräsvirta (1994) tests LM_{C3}, LM_{C2}, LM_{C1} reported in Panel C) of Table 5, we also executed a standard Chow test for the hypothesis of no structural break in the equilibrium correction terms of our model reported in Table 5, using dummy variables that allow for parameter shifts in 1989 and 1994. The test statistics suggested nonrejection of the null hypothesis of parameter constancy at conventional significance levels, confirming the evidence provided by the tests LM_{C3}, LM_{C2}, LM_{C1}.

from the funds rate target on the previous day. [21] Alternatively, deviations of the funds rate from the target level might be the result of forecast errors in implementing monetary policy (e.g., Thornton, 2001). Either way, at the daily frequency, the funds rate would tend to revert to the target level and, therefore, the TB. The reversion of the funds rate to the target would then appear in the data as the funds rate adjusting to the equilibrium relationship with the TB.

If the relatively fast adjustment of the funds rate is due to the Fed targeting the funds rate, one would expect to see a reduction in the speed at which the funds rate adjusts to the long-run equilibrium if the Fed's funds rate target is explicitly accounted for. Consequently, as a robustness check, we re-estimate our nonlinear asymmetric model including $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$ as a right-hand-side variable to explain movements in the FF. If the rapid adjustment of the funds rate is due to the Fed's targeting procedure, it seems plausible that $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$ should account for a significant fraction of the adjustment of the funds rate.

Also, if more rapid adjustment of the funds rate with daily data is due to the Fed targeting the funds rate, the results should change when lower frequency data are used. Hence, the model is estimated using monthly data over the period 1974–1999 to investigate this possibility.

Finally, despite its wide use in empirical analyses in economics and finance, the 3-month TB has characteristics that make it somewhat unique (e.g., see Duffee, 1996). Consequently, it is important to test whether the results are robust to the use of other 3-month interest rates. This is done by estimating the model using the daily 3-month CD rate in place of the TB. The CD rate was selected because large money-market banks finance part of their loan portfolios in the overnight federal funds market and with CDs. Consequently, federal funds and CDs represent alternative sources of funds for these banks. [22]

5. Robustness results

In this section we report several robustness checks carried out in order to evaluate the sensitivity of the empirical results reported in Section 4. In the first model (M1), $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$ is included on the right-hand-side of the nonlinear asymmetric ECM (7) and (8). [23] Data on the funds rate target are not available over the entire

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[21] See Taylor (2001) for a model where the supply of reserves is a function of the spread between the funds rate and the funds rate target on the previous day. See also Orphanides (2001) for a critique of this model and Thornton (2001) for an alternative rationale for the Fed's reaction to the previous day's spread between the funds rate and the funds rate target and for evidence of the extent to which the Fed responds to the previous day's spread between the funds rate and the funds rate target.

[22] A referee suggested that daily data are simply at too low a frequency to pin down the true adjustment process. The T-bill market is very liquid, so that adjustments may occur within minutes or so of a shock. In contrast, the federal funds market is a thinner and less liquid market. Consequently, the adjustment of the funds rate to shocks takes longer. The referee suggests that the finding that FF does most of the adjustment may be partly due to microstructural differences between the two markets.

[23] In fact, we allowed for longer (up to five) lags of $\Delta(\text{FF} - \text{FF}^{\text{target}})_t$, but lagged values from 2 to 5 were never found to be statistically significant at conventional significance levels.

sample period. In this case the model is estimated over the period 22 November, 1989 to 31 December, 1999, a period for which there is agreement that the Fed was explicitly targeting the FF. [24] In the second model (M2), Eqs. (7) and (8) are estimated using monthly data over the period from January 1974 to December 1999. In the third model (M3), the robustness of the findings are investigated by replacing TB_t with CD_t. [25]

The results of these robustness checks are summarized in Table 6, where, to save space, we report only the estimated equilibrium correction coefficients and some test statistics. In particular, Table 6 reports the equilibrium correction terms and the parameters associated with $\Delta(FF - FF^{\text{target}})_{t-1}$ (namely κ_1, linear, and κ_2, nonlinear, in the equation for ΔFF and the corresponding parameters, ϱ_1 and ϱ_2, in the equation for ΔTB). The linear coefficient on $\Delta(FF - FF^{\text{target}})_{t-1}$ is correctly signed and statistically significant in the funds rate equation. The coefficient is large in absolute value, suggesting that the funds rate tends to adjust rather quickly to deviations from the target rate. More importantly, the equilibrium correction term in the funds rate equation decreases when the funds rate target is explicitly accounted for, suggesting that some of the adjustment process for the funds rate reported in Table 5 was due to the reversion of the funds rate toward the funds rate target. Nevertheless, even allowing for this possibility, the funds rate continues to bear the brunt of the adjustment process. Moreover, the speed of adjustment of the TB is relatively slow and the coefficient on $\Delta(FF - FF^{\text{target}})_{t-1}$ is insignificantly different from zero. While some of the adjustment of the FF at the daily frequency appears to be due to reversion of the funds rate to the funds rate target, the conclusion that the FF bears most of the burden of the adjustment process is not overturned.

The results in the M2 column of Table 6 indicate that the conclusion that the funds rate bears the burden of the adjustment process is also robust to the frequency of the data. The estimated speed of adjustment of the funds rate is more than 20 times faster than that of the TB when monthly data are used. [26] Hence, consistent with the results reported in the M1 column, the finding that the funds rate bears the burden of the equilibrium adjustment process does not appear to be

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[24] We chose 22 November, 1989 to conform with Hamilton and Jordá (2001).

[25] In testing for cointegration between FF and TB using monthly data (as a preliminary to estimating M2) and between TB and CD (as a preliminary to estimating M3), in each case, the Johansen procedure suggested one significant eigenvalue and, therefore, one cointegrating relationship. Also, the cointegrating relationships were found to be stable over time using recursive estimation methods (results not reported to conserve space but available upon request). Nevertheless, as in the case of the relationship between the funds rate and the TB discussed in Section 4, the asymmetric ECM was unable to fully capture the nonlinearity in the underlying relationship and linearity tests indicated a model of the form (7) and (8).

[26] Also, note that the cointegrating parameter obtained using the Johansen procedure in a VAR comprising the FF rate and the TB rate at monthly frequency was very close to that reported in Table 2. This suggests that the finding of a cointegrating parameter that is not equal to the unity parameter implied by the EH may not be due to the inferential problems caused by the high volatility displayed by the daily time series (see Hall et al., 1996).

Table 6
Robustness: The magnitude of the nonlinear equilibrium correction coefficients

	M1	M2	M3
(A) Nonlinear ECM for ΔFF (M1, M2 and M3)			
ζ_1^- [linear u_{t-1}^-]	–	–	–
ζ_1^+ [linear u_{t-1}^+]	–	–	–
χ_1 [linear $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$]	−0.741 (0.025)	–	–
ζ_2^- [nonlinear u_{t-1}^-]	−0.153 (0.027)	−0.520 (0.151)	−0.324 (0.019)
ζ_2^+ [nonlinear u_{t-1}^+]	−0.032 (0.016)	−0.205 (0.033)	−0.087 (0.017)
χ_2 [nonlinear $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$]	–	–	–
J − ECM	$\{5.7 \times 10^{-7}\}$	$\{9.5 \times 10^{-6}\}$	$\{2.7 \times 10^{-81}\}$
Asymmetry	$\{3.4 \times 10^{-12}\}$	$\{4.7 \times 10^{-8}\}$	$\{4.4 \times 10^{-22}\}$
LR	$\{0.740\}$	$\{0.782\}$	$\{0.858\}$
(B) Nonlinear ECM for ΔTB (M1 and M2) and ΔCD (M3)			
ξ_1^- [linear u_{t-1}^-]	0.013 (0.003)	0.025 (0.011)	0.013 (0.004)
ξ_1^+ [linear u_{t-1}^+]	3.7×10^{-3} (3.0×10^{-3})	0.008 (0.003)	0.007 (0.001)
ϱ_1 [linear $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$]	–	–	–
ξ_2^- [nonlinear u_{t-1}^-]	–	–	–
ξ_2^+ [nonlinear u_{t-1}^+]	–	–	–
ϱ_2 [nonlinear $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$]	–	–	–
J − ECM	$\{4.5 \times 10^{-4}\}$	$\{0.035\}$	$\{0.012\}$
Asymmetry	$\{5.7 \times 10^{-9}\}$	$\{7.1 \times 10^{-7}\}$	$\{4.6 \times 10^{-4}\}$
LR	$\{0.348\}$	$\{0.593\}$	$\{0.590\}$

Notes: The model estimated is a nonlinear asymmetric ECM of the form:

$$\Delta Y_t = v_1 + \zeta_1^- u_{t-1}^- + \zeta_1^+ u_{t-1}^+ + \sum_{i=1}^{p} \mu_{1i} \Delta Y_{t-i} + \sum_{i=1}^{p} \mu_{2i} \Delta X_{t-i} + \left[v_2 + \zeta_2^- u_{t-1}^- + \zeta_2^+ u_{t-1}^+ \sum_{i=1}^{p} \pi_{1i} \Delta Y_{t-i} \right.$$
$$\left. + \sum_{i=1}^{p} \pi_{2i} \Delta X_{t-i} \right] \times \left\{ 1 - \exp\left[-\chi(u_{t-d} - v_3)^2 \right] \right\} + \text{innovations},$$

$$\Delta X_t = w_1 + \xi_1^- u_{t-1}^- + \xi_1^+ u_{t-1}^+ + \sum_{i=1}^{p} \vartheta_{1i} \Delta Y_{t-i} + \sum_{i=1}^{p} \vartheta_{2i} \Delta X_{t-i} + \left[w_2 + \xi_2^- u_{t-1}^- + \xi_2^+ u_{t-1}^+ \sum_{i=1}^{p} \phi_{1i} \Delta Y_{t-i} \right.$$
$$\left. + \sum_{i=1}^{p} \phi_{2i} \Delta X_{t-i} \right] \times \left\{ 1 - \exp\left[-\psi(u_{t-d} - w_3)^2 \right] \right\} + \text{innovations},$$

where Y and X are the two interest rates whose dynamic relationship is being modelled; v_1 and w_1 are constant terms; ζ_j^-, ζ_j^+, ξ_j^- and ξ_j^+ for $j = 1, 2$ are speed-of-adjustment coefficients; u_{t-1}^- and u_{t-1}^+ are negative and positive deviations from the long-run equilibrium involving Y and X (estimated using the Johansen procedure). Y and X are equal to FF and TB in model M1 and M2. M2 is estimated using monthly data (thus M2 is the same as the model (7) and (8) reported in Table 5, except that M2 is estimated using end-of-period monthly data rather than daily data). M1 is estimated using daily data but both ECM equations are augmented by adding $\Delta(\text{FF} - \text{FF}^{\text{target}})_{t-1}$ as a right-hand-side variable both in the linear component of the model (with associated coefficients $\varkappa_1$ and ϱ_1 respectively) and in the nonlinear component of the model (with associated coefficients $\varkappa_2$ and ϱ_2, respectively). In M3, Y and X are FF and CD, and the model is estimated using daily data. The reported estimated coefficients were obtained by NFIML estimation and applying a general to specific procedure; figures in parentheses denote estimated standard errors; figures in braces denote p-values. In the first column, in square brackets we report the variable associated with a particular coefficient, specifying whether it belongs to the linear or nonlinear

Table 6 (*continued*)

component of the model. $J - \text{ECM}$ is a test of joint significance of the equilibrium correction coefficients not restricted to zero in the final estimation for each equation, and is distributed as $\chi^2(m)$ (m being the number of restrictions) under the null; Asymmetry is a test of the null hypothesis of no asymmetric equilibrium correction; LR is a likelihood ratio test for the validity of the restrictions imposed in the final estimation.

critically dependent on the high-frequency adjustment of the funds rate to the funds rate target.

Comparing the result of M3 in Table 6 with the results in Table 5 reveals that this finding also does not depend on the choice of the 3-month rate. The funds rate continues to bear the burden of the adjustment process when the CD rate is used instead of the TB. Indeed, if anything, the estimated speed of adjustment of the funds rate is faster with the CD rate. Whatever accounts for the relatively rapid adjustment of the funds rate, it does not appear to be associated with the default-risk-free characteristic or favorable tax treatment (or other idiosyncratic characteristics, e.g., Duffee, 1996) of T-bills. [27]

6. Conclusion

This article examines the dynamic relationship between two key US money market interest rates, the FF and the 3-month TB, employing a very general nonlinear asymmetric vector ECM and using daily data over the period 1974–1999. The empirical results provide strong evidence of a cointegrating relationship between the FF and the TB that is remarkably stable over regimes of FF and monetary aggregate targeting. The conventional linear ECM is rejected when tested against the nonlinear ECM, indicating that the dynamic relationship between these rates is nonlinear. Moreover, consistent with the findings of Enders and Granger (1998), the adjustment to the equilibrium rate structure is asymmetric: the adjustment to negative deviations from the equilibrium rate structure is much faster than that for positive deviations.

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[27] At the suggestion of one of the referees we conducted a check of the robustness of our findings by assuming a reasonable state and local tax rate for the FF. To this end, we estimated the model assuming a constant tax rate ranging from 2% to 15%. The results were relatively insensitive to the tax rate except for a tax rate between 11% and 12%. For a tax rate in this range, the cointegrating parameter on the TB is indistinguishable from the theoretical parameter of minus unity—the constant term is then about 0.4. All of the other results, however, were qualitatively unchanged. Given that individuals cannot participate directly in the federal funds market, the corporate tax rate should be used. Most states have nominal marginal corporate tax rates that are well below 11% and some states have no corporate income taxes. Moreover, state and local income taxes account for a relatively small percentage of state and local government tax revenue. Consequently, the effective marginal corporate tax rate should be smaller than the average nominal tax rate. This experiment leads us to believe that the differential tax treatment of interest income on federal funds and T-bills is likely not to be the most important factor able to explain the results obtained here.

Although we provide a conjecture that could explain to some extent this asymmetry, we know of no convincing theoretical explanation for this result; however, this empirical regularity deserves further investigation.

Also, consistent with a number of previous studies, we find that the estimated cointegrating vector is inconsistent with the strict version of the conventional expectation hypothesis of the term structure of interest rates. This is not surprising. Empirical support for the strict version of the EH is generally weak (e.g., Campbell et al., 1997). Indeed, the support is particularly weak when the short-term rate is the FF (see, inter alia, Simon, 1990, 1994; Roberds et al., 1996; Thornton, 2002).

The more surprising result was the finding that the funds rate adjusts more rapidly than the TB toward the long-run equilibrium linking these two rates. The conventional view of monetary policy is that the Fed controls the FF and all other short-term rates key off of it. In some sense, the FF is thought to ''anchor'' the short end of the term structure. Strictly speaking, this paradigm assumes the EH holds, but even if the strict EH does not hold, there is no reason to doubt that markets are forward looking. Hence, it is reasonable to assume that other rates key off of the funds rate, given the apparent ability of the Fed to control the funds rate.

One way to reconcile our finding with the conventional view of monetary policy comes from noting that the funds rate tends to stay close to the Fed's target for the funds rate. If the TB is set more or less equal to the market's expectation for the Fed's funds rate target, estimates using daily data would suggest that the funds rate is adjusting to the TB when in fact the funds rate is merely reverting to the funds rate target. Several robustness checks were undertaken to investigate this possibility. Specifically, deviations of the funds rate from the Fed's target on the previous day were included in the model. While the speed of adjustment of the funds rate was somewhat reduced when deviations of the funds rate from the Fed's target on the previous day were included, the fundamental result that the funds rate bears most of the burden of the adjustment process remained. Also, the funds rate continued to bear the burden of the adjustment process when monthly data were used. Consequently, the finding that it is the funds rate that does most of the adjusting toward equilibrium does not appear to be due to the reversion of the funds rate to the funds rate target. It is also not due to the choice of the 3-month rate, as evidenced by the fact the results were qualitatively unchanged when the CD rate was used in place of the TB. [28]

This result from our dynamic model can also be reconciled with the conventional view of the relationship between the FF and other short-term US interest rates if the market is able to make accurate forecasts of the future level of the funds rate. If the market correctly predicts changes in the funds rate, the TB (and other short-term interest rates) would move in advance of the funds rate. In such a circumstance, it would appear that the funds rate is adjusting to the TB when in fact the TB is adjusting to the expected future FF. There is some evidence, however, that interest

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[28] While our results may seem somewhat unexpected, they add to some related studies. Garfinkel and Thornton, 1995, for example, found that the FF did not contain information that was not contained in either the overnight repurchase rate or the three-month TB.

rates are relatively difficult to predict even when the Fed is explicitly targeting the funds rate. [29] Nevertheless, this possibility deserves further investigation.

The reader is cautioned that, while the model used here is very general and flexible, the results are based on bivariate interest rate comparisons. The US money market is very complex and involves a wide range of credit instruments. Much more work needs to be done to understand these relationships. Nevertheless, the evidence presented here suggests that the relationship between the FF and other short-term interest rates is considerably more complex than models of monetary policy or the EH suggest.

Acknowledgements

Appendix A. Testing for linearity and for parameter constancy

This appendix describes the procedure employed to test for parameter constancy and for linearity briefly discussed in Section 2 and employed in Section 4.

Assuming that a plausible transition variable is $\hat{u}_{t-d}$, the appropriate auxiliary regression for the linearity tests against a STR alternative, which is an important preliminary to the specification and estimation of the nonlinear system (7) and (8), is the following:

$$\hat{v}_t = \vartheta_0' \mathbf{A}_t + \vartheta_1' \mathbf{A}_t \hat{u}_{t-d} + \vartheta_2' \mathbf{A}_t \hat{u}_{t-d}^2 + \vartheta_3' \mathbf{A}_t \hat{u}_{t-d}^3 + \text{innovations}, \tag{A.1}$$

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[29] For example, Robertson and Thornton (1997), who investigated the predictive accuracy of a rule based on the federal funds futures rate from October 1988 through August 1997, found that the forecasting rule correctly predicted a target change at the one-month horizon only about one-third of the time. This finding is consistent with Rudebusch (2001), who finds very little predictability of short-term interest rates beyond a 1-month horizon. For evidence of the predictability of changes in the Fed's funds rate target at much shorter horizons, see Poole and Rasche (2000), Kuttner (2001), Poole et al. (2002) and Sarno et al. (2002).

where $\hat{v}_t$ is the estimated disturbance retrieved from the parsimonious ECM being tested for linearity (in the present context it is the residual from the each of the models reported in Table 3), and $\mathbf{A}_t$ denotes the vector of explanatory variables in that ECM (see Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998). The general test for linearity against STR is then the ordinary F-test of the null hypothesis:

$$H_{0L} : \boldsymbol{\vartheta}'_1 = \boldsymbol{\vartheta}'_2 = \boldsymbol{\vartheta}'_3 = \mathbf{0} \tag{A.2}$$

for $d \in \{1, \ldots, D\}$, where $\mathbf{0}$ is a null vector. If linearity is rejected for more than one value of d, then d is determined as the value $(\hat{d})$ which minimizes the p-value of the linearity test, and we set $d = \hat{d}$.

The choice between a LSTR and an ESTR model is based on a sequence of nested tests within (A.2). First, the null hypothesis H_{0L} in (A.2) must be rejected using an ordinary F-test (F_L). Then the following hypotheses are tested:

$$H_{03} : \boldsymbol{\vartheta}'_3 = \mathbf{0}, \tag{A.3}$$

$$H_{02} : \boldsymbol{\vartheta}'_2 | \boldsymbol{\vartheta}'_3 = \mathbf{0}, \tag{A.4}$$

$$H_{01} : \boldsymbol{\vartheta}'_1 | \boldsymbol{\vartheta}'_2 = \boldsymbol{\vartheta}'_3 = \mathbf{0}. \tag{A.5}$$

Again, an F-test is used, with the corresponding test statistics denoted F_3, F_2, and F_1, respectively, and the decision rule is as follows: after rejecting H_{0L} in (A.2) and setting $d = \hat{d}$, the three hypotheses (A.3)–(A.5) are tested using F-tests; if the test of (A.4) has the smallest p-value, an ESTR is chosen, otherwise an LSTR is selected (see Granger and Teräsvirta, 1993; Teräsvirta, 1994, 1998). Of course, the linearity testing procedure is applied individually to both parsimonious equilibrium correction equations, namely the equation for ΔFF and the one for ΔTB.

This procedure is then easily extended to test for parameter stability, simply replacing the transition variable $\hat{u}_{t-d}$ with t and applying the same sequence of tests (see Lin and Teräsvirta, 1994). The test statistics correponding to F_3, F_2, and F_1 are termed LM_{C3}, LM_{C2}, and LM_{C1}, respectively.

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